

# **Resonance Behavior of Liquid Bridges under Axial and Lateral Oscillating Total Body Forces**

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## **Abstract**

Cylindrical liquid bridges that are magnetically levitated in air against gravity are subjected to either axial or lateral oscillations of the total body force. It was found that the frequency of the first resonance peak is maximum when the total body force is zero, and decreases with increasing total body force for both axial and lateral oscillations. The results are examined theoretically.

Fluid zones supported by one, two, or more solid supports have long been of interest due to their ubiquitous occurrence in both technology and nature. An axially-symmetric bridge supported by two equal, coaxial disks at each end is a simple configuration that is ideal for the study of static properties [1,2,3,4], bridge collapse [1,2,3], and vibrational dynamics [4,5,6,7,8,9,10,11].

In recent years bridge dynamics have received growing attention. Studies have been made [2,12] on pendant droplets and liquid bridges to examine the return to equilibrium, as well as the dynamics of bridge breaking. For example, using the technique of stabilization with an axial electric field, Sankaran and Saville suddenly changed the electric field and optically monitored the collapsing bridge [13]. Other experiments have focused on the resonance behavior of bridges. For example, Langbein, *et al.* examined the resonance frequencies of an axially-driven bridge as a function of the bridge's length to diameter ratio in a space-borne microgravity environment [10]. Sanz [4] and Perales and Meseguer [11] studied bridge resonances in a density-matched Plateau tank using a mechanical drive, and Morse *et al.* [14] employed ultrasonic radiation pressure in a Plateau tank to study the resonances. Resonances of bridges subjected to lateral accelerations were studied by Martinez in a space-borne experiment [15] and by Sanz and Diez using a Plateau tank [9]. A particular difficulty associated with space-borne experiments, however, is the inability to controllably examine resonances in other than a zero Bond number environment. Although this problem can be circumvented on Earth in a Plateau tank, the surrounding fluid bath can strongly influence the mode structure of the bridge [4,9], and may also couple to the mechanical drive. To obviate many of these problems, we very recently demonstrated [3] that time-varying magnetic levitation may be used to investigate the dynamics of bridge collapse in air, without the complicating influence of a fluid bath. In this paper we show that an oscillating magnetic field can be used to examine the resonances of a fluid bridge, oriented either axially or laterally with respect to the gravitational force. Moreover, the d.c. Bond number at which the experiment is performed may also be controlled. Our central result is that for both configurations, the frequency of the first

resonance decreases with increasing Bond number. Experimental results are compared with the one-dimensional (1-D) slice model [1].

In the ensuing discussion it is convenient to define two dimensionless quantities:

The slenderness ratio  $\Lambda \approx L/d$ , where  $L$  is the bridge's length and  $d$  the diameter, and the Bond number  $B \approx \rho g d^2/4\sigma$ , where  $g$  is the gravitational acceleration,  $\sigma$  is the surface tension, and  $\rho$  is the fluid density. When placed in a spatially inhomogeneous magnetic field  $H$ , a material of magnetic susceptibility  $\chi$  (per unit mass) experiences a magnetic force per unit mass given by  $F_{\text{mag}} = \chi \nabla(H^2)$ . When  $F_{\text{mag}}$  is antiparallel to the gravitational force, the total body force (per unit mass) on the fluid is given by  $g_{\text{eff}} = g - \chi \nabla(H^2)$ . Note that the Bond number must also be redefined in the presence of a magnetic force, *viz.*,  $B_{\text{eff}} = \rho[g - \chi \nabla(H^2)]d^2/4\sigma$ . Thus, the total body force vanishes when  $\chi \nabla(H^2) = g$ , and the bridge feels an effective gravity-free environment. Under this condition the bridge is a perfect cylinder. For  $\chi \nabla(H^2) \neq g$  the bridge distorts under gravity, with a Bond number  $B_{\text{eff}} \neq 0$ .

To facilitate levitation of the fluid, a conventional laboratory electromagnet was equipped with Faraday pole pieces to achieve a uniform  $F_{\text{mag}}$ , *i.e.*, a uniform  $\nabla(H^2)$ , over a volume of order  $1 \text{ cm}^3$  [16]. A pair of identical coaxial cylindrical rods was inserted into the magnet, such that one of the rods was mounted on a translation stage to control the separation between the rods' ends. The ends of the rods, each of diameter  $d = 0.315 \text{ cm}$ , were machined with a wetting barrier. Bridges were formed using the strongly paramagnetic solution of 60% by weight  $\text{MnCl}_2 \cdot 4\text{H}_2\text{O}$  in water, whose density was measured to be  $\rho = 1.45 \text{ g cm}^{-3}$ , and whose surface tension was determined to be  $\sigma = 70 \pm 5 \text{ ergs cm}^2$  by the capillary rise method. The magnet current  $i$  was first adjusted so that  $g_{\text{eff}} = 0$ , corresponding to  $B_{\text{eff}} = 0$ , and a bridge of a given slenderness ratio  $\Lambda$  was formed. The same procedure was used for both vertical bridges subjected to axial gravity and horizontal bridges subjected to lateral gravity. The magnet current was then oscillated around its average value  $i_0$  at

frequency  $\omega$  with a small amplitude  $\delta i$  ( $\ll i_0$ ), so that the total magnet current  $i = i_0 + \delta i \sin \omega t$ . Thus the magnetic component of force on the bridge,  $F_{\text{mag}}$ , which is proportional to  $(i_0 + \delta i \sin \omega t)^2$ , has a d.c. term that is adjusted to cancel some or all of Earth's gravity. The resulting total body force  $g_{\text{eff}}$  contains a term proportional to  $2i_0\delta i \sin \omega t$ , which oscillates at frequency  $\omega$ . Note that the component at  $2\omega$  is kept to less than one-twentieth of the  $\omega$ -component of  $g_{\text{eff}}$ , and thus can be neglected. A similar situation obtains for the Bond number  $B_{\text{eff}}$ , for which there is a d.c. component  $B_{\text{eff}}^0$  and an oscillating component  $\delta B_{\text{eff}}$  that is proportional to  $2i_0\delta i \sin \omega t$ . It should be noted that the modulation amplitude  $\delta B_{\text{eff}}$  may exceed the dc Bond number  $B_{\text{eff}}^0$ , which may be as small as zero in a completely gravity-compensated environment.

The bridge was imaged using a boroscope and charge coupled device (CCD) camera. We first examined vertical bridges in which  $g_{\text{eff}}$  was directed axially. In a near  $B_{\text{eff}}^0 = 0$  environment, a cylindrical bridge of given slenderness ratio  $\Lambda$  was subjected to oscillations of the Bond number. The angular frequency  $\omega$  was varied over the range  $6 \text{ s}^{-1}$  to  $75 \text{ s}^{-1}$ , corresponding to approximately 1 to 12 Hz, while the magnitude of the Bond number oscillations was kept constant. Higher frequencies were not yet accessible with our current equipment. The corresponding physical oscillations of the bridge were measured by replaying the video image frame-by-frame, as seen in Fig. 1 for a bridge with slenderness ratio  $\Lambda = 2.66$  at  $B_{\text{eff}}^0 = 0$ . At the antinode, a position  $\frac{1}{4}L$  from the end, the bridge oscillates between a narrow neck and a large bulge. The oscillation amplitude  $\delta d$  is defined as  $\frac{1}{2}(d_{\text{max}} - d_{\text{min}})$ , where  $d_{\text{max}}$  is the maximum diameter and  $d_{\text{min}}$  is the minimum diameter at this position. As a function of driving frequency, the amplitude  $\delta d$  was found to exhibit a clear maximum, corresponding to the first resonant frequency; the squares in Fig. 2 show the experimental values of the dimensionless amplitude  $\delta d/d$  vs. both angular frequency  $\omega$  and the corresponding dimensionless frequency  $\omega_0$  [  $\cdot \omega \frac{\eta d}{2\sigma}$  ] for a bridge of slenderness ratio  $\Lambda = 2.66$  at

$B_{\text{eff}}^{\circ} = 0$ . The d.c. Bond number  $B_{\text{eff}}^{\circ}$  was then changed to a nonzero value, resulting in a sagging bridge. The position of the neck was approximately  $\frac{1}{4}L$  from the top of the bridge, as seen in Fig. 3a. The magnet current was again oscillated and the amplitude of oscillations was determined by the change of diameter at the bridge's narrowest point. Figs. 3b and 3c show these oscillations for a bridge at  $\Lambda = 2.66$  at  $B_{\text{eff}}^{\circ} = 0.03$ . The behavior of  $\delta d/d$  vs. driving frequency at three different Bond numbers  $B_{\text{eff}}^{\circ}$  for slenderness ratio  $\Lambda = 2.66$  is shown in Fig. 2, where the peak amplitude occurs at decreasing frequency for increasing Bond number. The experiment was repeated at several slenderness ratios, each at several d.c. Bond numbers. A monotonic decrease of resonance frequency was observed with increasing Bond number in every case, consistent with model calculations [17]. The solid points in Fig. 4 show the experimental first resonance frequencies  $\omega$  (as well as the corresponding dimensionless first resonance frequencies  $\omega_0$ ) vs. d.c. Bond number  $B_{\text{eff}}^{\circ}$  for four different slenderness ratios. A direct comparison with previously reported measurements [6,14] may be made at  $B_{\text{eff}}^{\circ} = 0$ ; the present results are found to be in good agreement with the earlier measurements.

Next we examined horizontal bridges that were subjected to lateral gravity. For these bridges the dimensionless oscillation amplitude was taken to be twice the maximum displacement of the outer edge of the bridge divided by the diameter  $d$  of the supporting rods, *i.e.*, the bridge diameter at  $B_{\text{eff}}^{\circ} = 0$ . The displacement measurement was made at  $\frac{1}{2}L$ , corresponding to the center of the bridge. Figs. 5a and 6a show undisturbed horizontal bridges of slenderness ratio  $\Lambda = 2.58$  at  $B_{\text{eff}}^{\circ} = 0$  and at  $B_{\text{eff}}^{\circ} = 0.065$ , respectively. Figs. 5b, 5c, 6b, and 6c show the maximum displacement of these bridges when driven near resonance. Fig. 7 shows the dimensionless oscillation amplitude vs. driving frequency for bridges of slenderness ratio  $\Lambda = 2.58$  at three different Bond numbers. Again the resonance frequencies were found to decrease with increasing

d.c. Bond number  $B_{\text{eff}}^{\circ}$ . Fig. 8 shows the first resonance frequencies of the horizontal bridges vs.  $B_{\text{eff}}^{\circ}$  at several different slenderness ratios. Notice that although both axial and lateral gravity bridges show a decrease of the first resonance frequency with increasing d.c. Bond number, the resonance frequencies exhibit a sharp decline at higher Bond number for the lateral gravity case. This reflects the close proximity of the lateral bridge to its stability limit. At  $B_{\text{eff}}^{\circ} = 0$  the results are in good agreement with previously reported measurements.

Due to the nonlinearity introduced by the free boundary motion, the numerical modeling of the response of a free liquid surface to a time-dependent residual acceleration is quite complicated. However, the model used by Zhang and Alexander [5] has been shown to give good results. They generalized a previous model due to Meseguer [1] to produce a nonlinear one-dimensional model for a viscous isothermal liquid bridge in which the effects of radial motion of the fluid were neglected. In this model the liquid bridge is held vertical and is subjected to an axial gravitational jitter. The liquid is assumed to be an isothermal incompressible Newtonian fluid. Additionally, it is assumed that internal fluid motion is caused *only* by capillary pressure gradients due to interface deformation in response to acceleration, that the effect of the atmosphere around the bridge may be neglected, and that only the response along the axial direction need be considered.

This approach is a simplification of the full three-dimensional set of equations for mass and momentum transfer in the liquid, the kinematic boundary condition, the balance of normal and tangential force components at the surface, the conditions at the support rods, the initial conditions of a cylinder and of zero fluid momentum, and the requirement of axisymmetry. Full details are given in Ref. 5, Eqs 1-10. These equations define an unsteady free boundary problem since the shape of the interface is *a priori* unknown and must be determined along with the velocities and pressure as part of the solution. By restricting the calculations to one dimension [1], we can greatly simplify the problem so as to examine a relatively wide range of parameters without excessive

computation time. This approach has been used in several previous numerical studies of bridge dynamics [1,5], and has been shown to provide physically reliable results as long as the slenderness ratio  $\Lambda$  is large, typically  $\Lambda > 2$ . It is based on a one-dimensional model for fluid jets [18] in which the axial velocity component is assumed to depend only on the  $z$ -coordinate and time. The resonance frequency of the liquid bridge for a given value of d.c. Bond number  $B_{\text{eff}}^{\circ}$  is found by computing the tolerable acceleration at which the liquid bridge breaks as a function of frequency. This approximation and the numerical method for the solution follow the approach used by Zhang and Alexander [5].

In Fig. 4 we plot the model results for bridges (open points) subjected to axial gravity at three slenderness ratios. Also shown by the solid line is the calculation by Meseguer [6] for a bridge of slenderness ratio  $\Lambda = 2.6$ . For the longest bridges studied, our model results quite accurately mimic the experimental resonance frequencies, although those of Ref. 6 are too high by about 10%. This discrepancy is consistent with comparisons made by Meseguer to observations of unintentionally oscillating microgravity bridges with very small  $B_{\text{eff}}^{\circ}$ . For smaller  $\Lambda$ , however, our calculated results are less consistent with the experimental data. Given the poor agreement at  $\Lambda = 2.22$ , no attempt was made to apply the model to the data at  $\Lambda = 2.06$ . Because the model does not consider the radial component of momentum, results are expected to be best for the longer bridges, as fluid flow along the bridge  $s$  axis dominates the behavior. For smaller  $\Lambda$ , where the radial dimensions become relatively more important, it is well known that the 1-D slice model breaks down [5]. We are currently developing code for modeling bridges subjected to lateral gravity; results will be published elsewhere [19].

Our results clearly demonstrate the efficacy of time-varying magnetic levitation as a tool for the study of bridge dynamics. Not only were we able to examine the first resonance of bridges in a terrestrial environment without a surrounding bath, but were also able to study the dynamics in the

presence of a controllable d.c. Bond number. At  $B_{\text{eff}}^0 = 0$  our data are consistent with extant results in the literature. Moreover, we found that the resonance frequency decreases with increasing  $B_{\text{eff}}^0$ ; this reflects the closer proximity of the bridge to its stability limit.

To summarize, we have examined the resonance behavior of liquid bridges in air. While it is well known that we can simulate microgravity conditions for static bridges using neutral buoyancy techniques [2,9], the use of neutral buoyancy precludes the study of bridge dynamics for bridges surrounded by air or some other gas. While it is true that *small* slenderness bridges can be examined without neutral buoyant encapsulation, our technique facilitates the study of oscillatory bridge dynamics to at least within 90% of the Rayleigh limit.

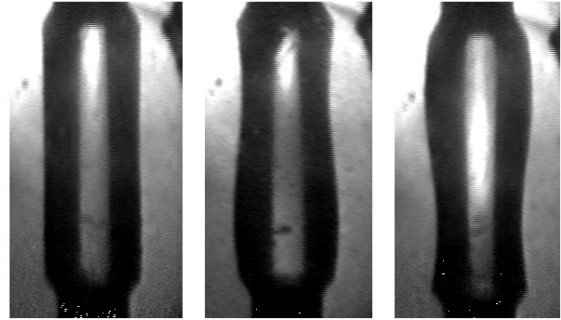
**Acknowledgments:** This work was supported by the National Aeronautics and Space Administration's Microgravity Program under Grants No. NAG8-1270 and NAG3-1864.

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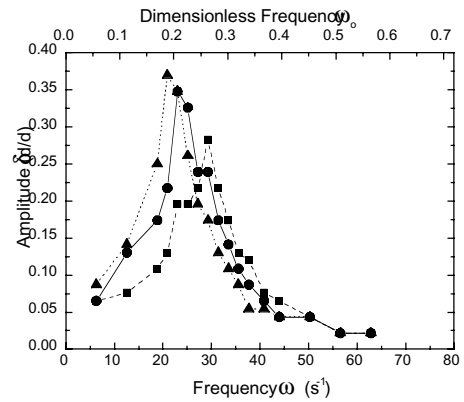
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**Figures**

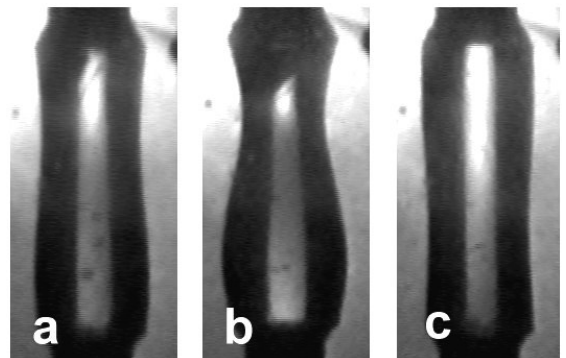
1. Bridge of slenderness ratio  $\Lambda = 2.66$  in a  $B_{\text{eff}}^0 = 0$  environment subjected to axial accelerations near its resonance frequency. a) Bridge with zero time-varying Bond number, *i.e.*,  $\delta B_{\text{eff}} = 0$ . b) and c) Maximum displacements for bridge subjected to  $\delta B_{\text{eff}} = 0.0245$ .



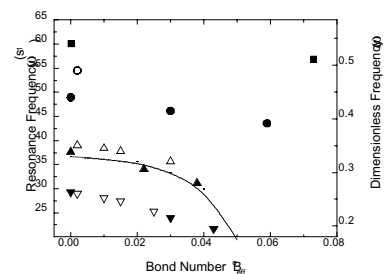
2. Relative oscillation amplitude (see text) vs. driving frequency for bridge of  $\Lambda = 2.66$  subjected to axial accelerations.  $\omega = 0$  corresponds to  $B_{\text{eff}}^0 = 0$ ,  $\omega = 0.029$  corresponds to  $B_{\text{eff}}^0 = 0.029$ , and  $\omega = 0.043$  corresponds to  $B_{\text{eff}}^0 = 0.043$ . Data points are connected by lines as a guide to the eye.



3. Same as Fig. 1 except that  $B_{\text{eff}}^0 = 0.03$ .



1. Resonance frequencies vs.  $B_{\text{eff}}^0$  for axially accelerated bridges. Closed points correspond to experimental results, and open points to numerical results.  $\omega = 0$  corresponds to  $\Lambda = 2.06$ ,  $\omega = 0.029$  correspond to  $\Lambda = 2.22$ ,  $\omega = 0.043$  correspond to  $\Lambda = 2.66$ .

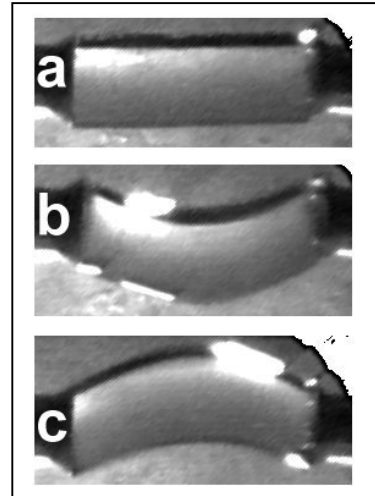


2.50, and  $\underline{\quad}$  correspond to  $\Lambda = 2.66$ . Solid line corresponds to numerical results for  $\Lambda = 2.6$  by Meseguer [6].

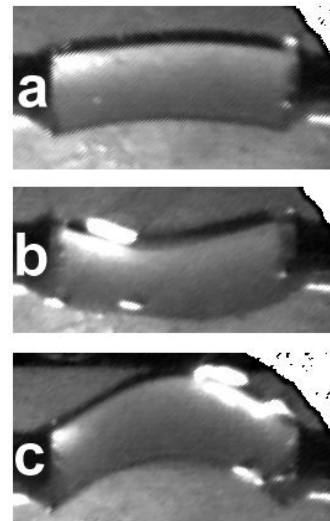
5. Bridge of slenderness ratio  $\Lambda = 2.58$  in a  $B_{\text{eff}}^0 = 0$  environment subjected to lateral accelerations near its resonance frequency.

a) Bridge with zero time-varying Bond number, *i.e.*,  $\delta B_{\text{eff}} = 0$ .

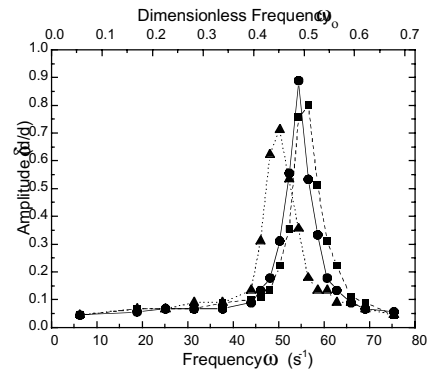
b) and c) Maximum displacements for bridge subjected to  $\delta B_{\text{eff}} = 0.0245$ .



6. Same as Fig. 5 except that  $B_{\text{eff}}^0 = 0.065$ . Note that for the largest amplitude oscillations the next mode is becoming excited.



7. Relative oscillation amplitude (see text) vs. driving frequency for bridge of  $\Lambda = 2.66$  subjected to lateral accelerations.  $\omega < \omega_0$  corresponds to  $B_{\text{eff}}^0 = 0$ ,  $\omega = \omega_0$  corresponds to  $B_{\text{eff}}^0 = 0.026$ , and  $\omega > \omega_0$  corresponds to  $B_{\text{eff}}^0 = 0.065$ . Data points are connected by lines as a guide to the eye.



1. Resonance frequencies vs.  $B_{\text{eff}}^0$  for laterally accelerated bridges.  $\omega < \omega_0$  corresponds to  $\Lambda = 2.33$ ,  $\omega = \omega_0$  corresponds to  $\Lambda = 2.40$ ,  $\omega > \omega_0$  corresponds to  $\Lambda = 2.58$ , and  $\omega > \omega_0$  corresponds to  $\Lambda = 2.77$ . Solid lines are drawn as a guide to the eye.

